Jawaban UTS 2 Analisis Real B
08 Juni 2009

1. $f(x) = \dfrac{1}{\sqrt{x}}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{x - (x+h)}{h\sqrt{x(x+h)}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{h\sqrt{x(x+h)}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{x(x+h)}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= -\frac{1}{\sqrt{x(x+0)}\left(\sqrt{x} + \sqrt{x+0}\right)} = -\frac{1}{\sqrt{x^2}\left(\sqrt{x} + \sqrt{x}\right)}$$

$$= -\frac{1}{2\sqrt{x}\sqrt{x^2}} = -\frac{1}{2\sqrt{x^3}} = -\frac{1}{2}x^{-3/2}$$

2. $h(x) = x^3 + 2x^2, \forall x \in \mathbb{R}, h^{-1}$ invers $h$ di $\mathbb{R}$.

$h^{-1}(y)$ yang berkorespondensi dg $x = -2, 0, 2$ masing-masing adalah:

$\dfrac{1}{h'(-2)} = \dfrac{1}{4}, \dfrac{1}{h'(0)} =$ tak terdefinisi, $\dfrac{1}{h'(2)} = \dfrac{1}{20}$

3. bukti $\left|Sin\ x - Sin\ y\right| \le \left|x - y\right|, \forall x, y \in \mathbb{R}$

$\forall x, y \in \mathbb{R}$ berlaku salah satu dari 3 kemungkinan:

i) $x < y$, maka menurut tr nilai rata-rata ada $c \in (x, y)$ sehingga

$Sin\ y - Sin\ x \le Cos\, c\,(y - x)$

$\left|Sin\ y - Sin\ x\right| \le \left|Cos\, c\right|\left|y - x\right|$

$\left|Sin\ x - Sin\ y\right| \le \left|Cos\, c\right|\left|x - y\right| \le \left|x - y\right|$, karena $0 \le \left|Cos\, c\right| \le 1$.

ii) untuk $x = y$ jelas kedua ruas akan bernilai 0.

iii) untuk $x > y$ caranya serupa dg i).

4. $f(x) = x|x| = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

akan diteliti apakah $f(x)$ dapat diturunkan di $x = 0$.

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{-x^2 - 0^2}{x - 0} = \lim_{x \to 0^-} -x = 0.$$

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{x^2 - 0^2}{x - 0} = \lim_{x \to 0^+} x = 0.$$

jadi dapat $f(x)$ diturunkan $\forall x \in \mathbb{R}$. Turunannya adalah

$f'(x) = \begin{cases} -2x, & x < 0 \\ 2x, & x \geq 0 \end{cases}$ atau dapat ditulis $f'(x) = 2|x|, \forall x \in \mathbb{R}$